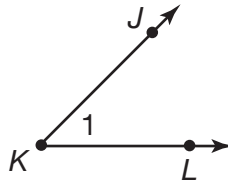


Angles



Getting the Idea

An **angle** is a geometric figure formed by two rays that have a common endpoint called the **vertex**. The angle below can be named $\angle 1$, $\angle JKL$, $\angle LKJ$, or $\angle K$.



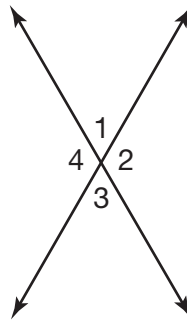
Angles are measured in degrees ($^\circ$) and can be classified by their angle measures.

An **acute angle** is an angle that measures less than 90° .

A **right angle** is an angle that measures 90° .

An **obtuse angle** is an angle that measures greater than 90° and less than 180° .

Intersecting lines can form some special angle pairs.



Adjacent angles are two angles with a side in common.

$\angle 1$ and $\angle 2$ are one set of adjacent angles.

Vertical angles are two non-adjacent angles, formed by intersecting lines, and are congruent.

$\angle 1$ and $\angle 3$ are one set of vertical angles.

Supplementary angles are two angles whose measures have a sum of 180° .

Supplementary angles that are adjacent angles form a **straight angle**.

$\angle 4$ and $\angle 3$ are one set of supplementary angles.

Complementary angles are two angles whose measures have a sum of 90° .

To write the measure of an angle, you can use the abbreviation, m.

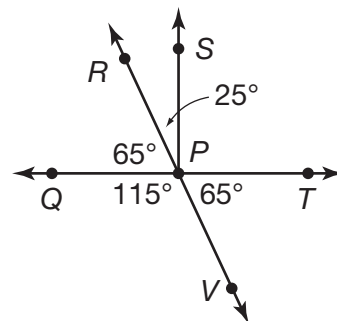
For example, “the measure of angle x” can be written as “m $\angle x$.”

Example 1

Look at the diagram on the right.

Find the following:

- a pair of complementary angles
- a pair of supplementary angles
- a pair of adjacent angles
- a pair of vertical angles



Strategy Use the definitions to identify the angle pairs.

Step 1

Find a pair of complementary angles.

Find two angles that have a sum of 90° .

$$m\angle QPR = 65^\circ \text{ and } m\angle RPS = 25^\circ.$$

$$65^\circ + 25^\circ = 90^\circ$$

Step 2

Find a pair of supplementary angles.

Find two angles that have a sum of 180° .

$$m\angle QPV = 115^\circ \text{ and } m\angle TPV = 65^\circ.$$

$$115^\circ + 65^\circ = 180^\circ$$

Note: Another pair of supplementary angles is $\angle QPR$ and $\angle QPV$.

Step 3

Find a pair of adjacent angles.

Find two angles with a side in common.

$\angle QPR$ and $\angle RPS$ have \overrightarrow{PR} in common.

Note: Other pairs of adjacent angles are $\angle RPS$ and $\angle SPT$, $\angle SPT$ and $\angle TPV$, $\angle TPV$ and $\angle QPV$, and $\angle QPV$ and $\angle QPR$.

Step 4

Find a pair of vertical angles.

Find two non-adjacent angles formed by intersecting lines.

\overrightarrow{RV} and \overrightarrow{QT} intersect at point P to form $\angle QPR$ and $\angle TPV$.

These angles are vertical angles, and they are congruent.

Solution $\angle QPR$ and $\angle RPS$ are a pair of complementary angles.

$\angle QPV$ and $\angle TPV$ are a pair of supplementary angles.

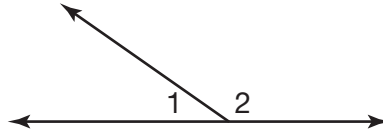
$\angle QPR$ and $\angle RPS$ are a pair of adjacent angles.

$\angle QPR$ and $\angle TPV$ are a pair of vertical angles.

Example 2

In the figure, the measure of $\angle 1$ is 35° .

What is the measure of $\angle 2$?



Strategy Look for a special angle pair.

Step 1

Decide what type of angles are $\angle 1$ and $\angle 2$.

The angles are adjacent angles that form a straight line.

So, $\angle 1$ and $\angle 2$ are supplementary angles.

Step 2

What angle measures do you know?

The measure of $\angle 1$ is 35° .

Supplementary angles have a sum of 180° .

Step 3

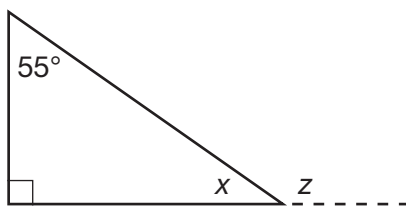
Subtract 35° from 180° to find the measure of $\angle 2$.

$$180^\circ - 35^\circ = 145^\circ$$

Solution The measure of $\angle 2$ is 145° .

Example 3

What are the measures of $\angle x$ and $\angle z$?



Strategy Look for special relationships between angles.

Step 1

Find the measure of $\angle x$.

The measures of the angles in a triangle have a sum of 180° .

The measures given are 55° and a right angle, which measures 90° .

Write an equation and solve for the measure of $\angle x$.

$$55^\circ + 90^\circ + m\angle x = 180^\circ$$

$$145^\circ + m\angle x = 180^\circ$$

$$145^\circ - 145^\circ + m\angle x = 180^\circ - 145^\circ$$

$$m\angle x = 35^\circ$$

Step 2

Identify the angle relationship of $\angle x$ and $\angle z$.

$\angle x$ is adjacent to $\angle z$. The two angles form a straight line.

Supplementary angles that are adjacent angles form a straight line.

So, $\angle x$ and $\angle z$ are supplementary angles.

Step 3

Recall the definition of supplementary angles.

Supplementary angles have a sum of 180° .

Step 4

Write an equation to find the measure of $\angle z$.

$$m\angle x + m\angle z = 180^\circ$$

$$35^\circ + m\angle z = 180^\circ$$

$$35^\circ - 35^\circ + m\angle z = 180^\circ - 35^\circ$$

$$m\angle z = 145^\circ$$

Solution The measure of $\angle x$ is 35° . The measure of $\angle z$ is 145° .



Coached Example

Two angles are complementary angles. One angle measures 59° .

The other angle measures $(4n - 1)^\circ$. What is the value of n ?

Complementary angles have a measure of _____.

The measure of one angle is _____, and the measure of the other angle is _____.

Write an equation for the total of the two angles.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 90$$

Solve the equation for n .

To find the measure of the unknown angle, substitute the value of n into $4n - 1$ and evaluate. _____

Check that the sum of the angle measures is 90° .

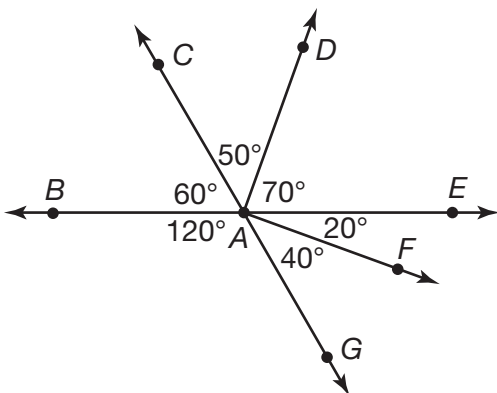
The value of n is _____.



Lesson Practice

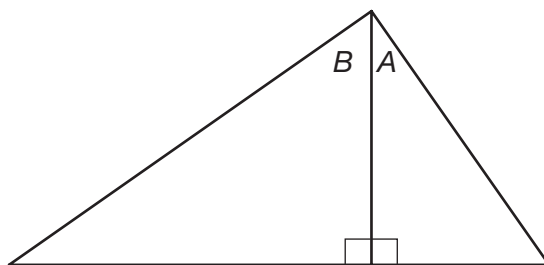
Choose the correct answer.

Use the diagram for questions 1–4.



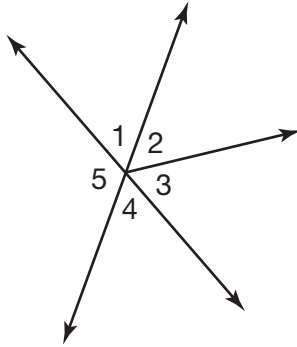
- Which angle is complementary to $\angle FAE$?
 - $\angle GAF$
 - $\angle BAC$
 - $\angle DAE$
 - $\angle DAC$
- Which angle is supplementary to $\angle GAB$?
 - $\angle BAC$
 - $\angle DAC$
 - $\angle DAE$
 - $\angle EAF$
- Which pair of angles are adjacent angles?
 - $\angle BAC$ and $\angle GAF$
 - $\angle DAC$ and $\angle GAF$
 - $\angle DAE$ and $\angle BAC$
 - $\angle CAD$ and $\angle DAE$

- Which pair of angles are vertical angles?
 - $\angle EAG$ and $\angle BAC$
 - $\angle GAB$ and $\angle DAF$
 - $\angle DAC$ and $\angle DAF$
 - $\angle BAC$ and $\angle CAE$
- $\angle A$ and $\angle B$ are complementary angles. If $m\angle A$ is 35° , what is the measure of $\angle B$?



- 55°
 - 65°
 - 90°
 - 145°
- $\angle J$ and $\angle K$ are supplementary angles. If $m\angle K$ is 84° , what is the measure of $\angle J$?
 - 16°
 - 86°
 - 96°
 - 106°

Use the diagram for questions 7 and 8.



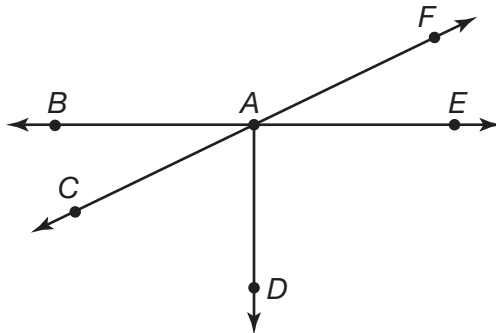
7. If $m\angle 1$ is 48° , what is the measure of $\angle 4$?

- A. 42°
- B. 48°
- C. 58°
- D. 132°

8. If $m\angle 4 = 2x^\circ$ and $m\angle 5 = 4x^\circ$, what is the value of x ?

- A. 20
- B. 30
- C. 60
- D. 120

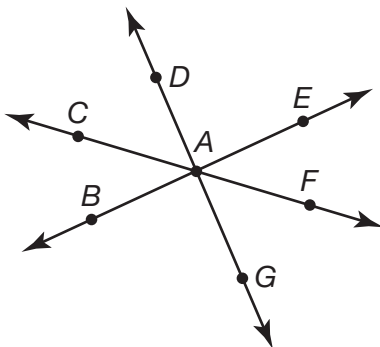
9. In the diagram below, $\angle DAE$ is a right angle and $m\angle BAF = 155^\circ$.



A. What is the measure of $\angle BAC$? Explain your answer.

B. What are the measures of $\angle FAE$ and $\angle CAD$? Explain your answers.

10. Select True or False for each statement.



- A. $\angle GAF$ and $\angle FAD$ are adjacent angles. True False
- B. $\angle DAC$ and $\angle EAF$ are adjacent angles. True False
- C. $\angle BAF$ and $\angle CAE$ are vertical angles. True False
- D. $\angle CAB$ and $\angle EAF$ are vertical angles. True False

11. $\angle C$ and $\angle D$ are complementary. The measure of $\angle D$ is 48° . Circle the number that makes the statement true.

The measure of $\angle C$ is

41

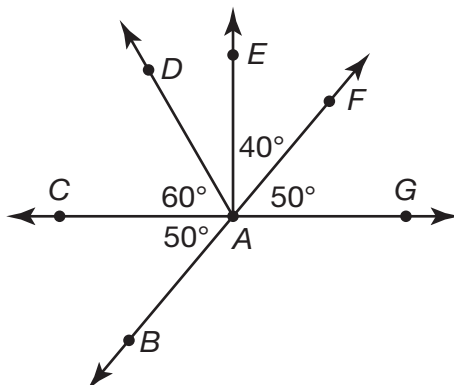
42

47

48

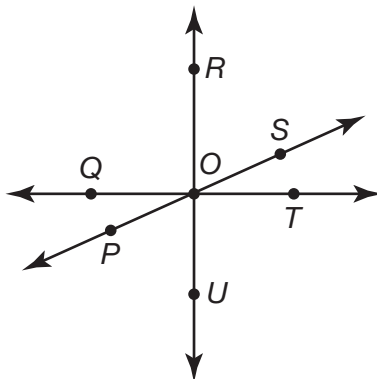
$^\circ$.

12. Select True or False for each statement.



- | | | | |
|----|---|----------------------------|-----------------------------|
| A. | $m\angle DAE$ is 30° . | <input type="radio"/> True | <input type="radio"/> False |
| B. | $m\angle DAE$ is 40° . | <input type="radio"/> True | <input type="radio"/> False |
| C. | $\angle GAF$ and $\angle FAE$ are supplementary angles. | <input type="radio"/> True | <input type="radio"/> False |
| D. | $\angle CAE$ and $\angle EAG$ are supplementary angles. | <input type="radio"/> True | <input type="radio"/> False |
| E. | $\angle BAD$ and $\angle DAF$ are supplementary angles. | <input type="radio"/> True | <input type="radio"/> False |

13. In the figure below, $m\angle ROT = 90^\circ$. Draw a line from each angle type to its angle pair.



- | | | | |
|----|-----------------|-----------------------|-------------------------------|
| A. | acute angles | <input type="radio"/> | $\angle QOP$ and $\angle SOT$ |
| B. | right angles | <input type="radio"/> | $\angle ROT$ and $\angle ROQ$ |
| C. | obtuse angles | <input type="radio"/> | $\angle POR$ and $\angle POT$ |
| D. | vertical angles | <input type="radio"/> | $\angle SOT$ and $\angle ROS$ |